EXPONENT RULES & PRACTICE

1. **PRODUCT RULE:** To multiply when two bases are the same, write the base and ADD the exponents.

$$x^m \cdot x^n = x^{m+n}$$

Examples:

A. $x^3 \cdot x^8 = x^{11}$ B. $2^4 \cdot 2^2 = 2^6$ C. $(x^2y)(x^3y^4) = x^5y^5$

2. **QUOTIENT RULE:** To divide when two bases are the same, write the base and SUBTRACT the exponents.

$$\frac{x^m}{x^n} = x^{m-n}$$

Examples:

A. $\frac{x^5}{x^2} = x^3$ B. $\frac{3^5}{3^3} = 3^2$ C. $\frac{x^2y^5}{xy^3} = xy^2$

3. **ZERO EXPONENT RULE:** Any base (except 0) raised to the zero power is equal to one.

$$x^0 = 1$$

Examples:

A. $y^0 = 1$ B. $6^0 = 1$ C. $(7a^3b^{-1})^0 = 1$

4. **POWER RULE:** To raise a power to another power, write the base and MULTIPLY the exponents.

$$(x^m)^n = x^{m \cdot n}$$

Examples:

A. $(x^3)^2 = x^6$ B. $(3^2)^4 = 3^8$ C. $(z^5)^2 = z^{10}$

5. **EXPANDED POWER RULE:**

$$(xy)^m = x^m y^n \qquad \left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$$

Examples:

A. $(2a)^3 = 2^3a^3 = 8a^3$

B. $(6x^3)^2 = 6^2(x^3)^2 = 36x^6$

C. $\left(\frac{x^2}{y}\right)^4 = \frac{(x^2)^4}{y^4} = \frac{x^8}{y^4}$

D. $\left(\frac{2x}{3y^2}\right)^3 = \frac{(2x)^3}{(3y^2)^3} = \frac{2^3x^3}{3^3(y^2)^3} = \frac{8x^3}{27y^6}$

6. **NEGATIVE EXPONENTS:** If a factor in the numerator or denominator is moved across the fraction bar, the sign of the exponent is changed.

$$x^{-m} = \frac{1}{x^m} \qquad \frac{1}{x^{-m}} = x^m \qquad \left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$$

Examples:

A. $x^{-3} = \frac{1}{x^3}$ B. $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$ C. $-4x^5y^{-2} = \frac{-4x^5}{y^2}$

D. $\left(\frac{x^2}{y}\right)^{-3} = \left(\frac{y}{x^2}\right)^3 = \frac{y^3}{x^6}$ E. $(3x^{-2}y)(-2xy^{-3}) = -6x^{-1}y^{-2} = \frac{-6}{xy^2}$

F. $\frac{a^{-2}b^3}{c^{-4}d^{-1}} = \frac{b^3c^4d}{a^2}$ G. $(-2x^2y^{-4})^{-2} = \left(\frac{-2x^2}{y^4}\right)^{-2} = \left(\frac{y^4}{-2x^2}\right)^2 = \frac{y^8}{4x^4}$

CAUTION: $-x \neq \frac{1}{x}$ For example: $-3 \neq \frac{1}{3}$

REMEMBER: An exponent applies to <u>only</u> the factor it is directly next to ***unless*** parentheses enclose other factors.

Examples:

A. $(-3)^2 = (-3)(-3) = 9$ B. $-3^2 = -9$

EXPONENTS PRACTICE

Simplify:

1. $3 \cdot 4^3$
2. $4x^3 \cdot 2x^3$
3. $x^5 \cdot x^3$
4. $2x^3 \cdot 2x^2$
5. $\frac{6^5}{6^3}$
6. $\frac{x^4}{x^7}$
7. 8^0
8. $-(9x)^0$
9. $(y^4)^3$
10. $(x^2y)^4$
11. $\frac{6x^7}{2x^4}$
12. $\frac{8x^5}{4x^2}$
13. $(2cd^4)^2(cd)^5$
14. $(2fg^4)^4(fg)^6$
15. $\frac{x^5y^6}{xy^2}$
16. $\frac{x^2y^5}{xy^4}$
17. $\left(\frac{4x^5y}{16xy^4}\right)^3$
18. $\left(\frac{5x^3y}{20xy^5}\right)^4$
19. y^{-7}
20. 7^{-2}
21. $\frac{1}{x^{-5}}$
22. $\frac{1}{2^{-4}}$
23. $x^5 \cdot x^{-1}$
24. x^{-6}
25. $x^9 \cdot x^{-7}$
26. $(j^{-13})(j^4)(j^6)$
27. $\frac{x^{-1}}{x^{-8}}$
28. $\frac{52x^6}{13x^{-7}}$
29. $f^{-3}(f^2)(f^{-3})$
30. $\frac{x^{-4}}{x^{-9}}$
31. $\frac{24x^6}{12x^{-8}}$
32. $\frac{3x^2y^{-3}}{12x^6y^3}$
33. $(2x^3y^{-3})^{-2}$
34. $\frac{2x^4y^{-4}}{8x^7y^3}$
35. $(4x^4y^{-4})^3$
36. $5x^2y(2x^4y^{-3})$
37. $\left(\frac{-7a^2b^3c^0}{3a^3b^4c^3}\right)^{-4}$
38. $\left(\frac{-2a^3b^2c^0}{3a^2b^3c^7}\right)^{-2}$

EXPONENTS PRACTICE ANSWERS

1. 192
2. $8x^6$
3. x^8
4. $4x^5$
5. 36
6. $\frac{1}{x^3}$
7. 1
8. -1
9. y^{12}
10. x^8y^4
11. $3x^3$
12. $2x^3$
13. $4c^7d^{13}$
14. $16f^{10}g^{22}$
15. x^4y^4
16. xy
17. $\frac{x^{12}}{64y^9}$
18. $\frac{x^8}{256y^{16}}$
19. $\frac{1}{y^7}$
20. $\frac{1}{49}$
21. x^5
22. 16
23. x^4
24. $\frac{1}{x^6}$
25. x^2
26. $\frac{1}{j^3}$
27. x^7
28. $4x^{13}$
29. $\frac{1}{f^4}$
30. x^5
31. $2x^{14}$
32. $\frac{1}{4x^4y^6}$
33. $\frac{y^6}{4x^6}$
34. $\frac{1}{4x^3y^7}$
35. $\frac{64x^{12}}{y^{12}}$
36. $\frac{10x^6}{y^2}$
37. $\frac{81a^4b^4c^{12}}{2401}$
38. $\frac{9b^2c^{14}}{4a^2}$

4.6 Applying Exponent Laws – including negative and rational exponents (p. 238)

Examples #1

Simplify by writing as a single power. Use BEDMAS to remember what order to follow in simplifying the expression. At the same time, remember and apply all exponent laws, including law of negative exponents. Write final answer with positive exponents.

a) $-0.8^2 \cdot 0.8^{-7}$

b) $\left[\left(-\frac{4}{5}\right)^2\right]^{-3} \div \left[\left(-\frac{4}{5}\right)^4\right]^{-5}$

c) $\frac{\left(1.5^{-3}\right)^{-5}}{1.5^5}$

d) $\frac{9^{\frac{5}{4}} \cdot 9^{-\frac{1}{4}}}{9^{\frac{3}{4}}}$

e) $m^4n^{-2} \cdot m^2n^3$

f) $\frac{6x^4y^{-3}}{14xy^2}$

g) $\left(25a^4b^2\right)^{\frac{3}{2}}$

Try these

a) $\left(x^3 y^{-\frac{3}{2}}\right)\left(x^{-1} y^{\frac{1}{2}}\right)$ b) $\dfrac{12x^{-5}y^{\frac{5}{2}}}{3x^{\frac{1}{2}}y^{-\frac{1}{2}}}$ c) $\left(\dfrac{50x^2y^4}{2x^4y^7}\right)^{\frac{1}{2}}$

Identify the error in each solution.

a) $\dfrac{12x^{-5}y^{\frac{5}{2}}}{3x^{\frac{1}{2}}y^{-\frac{1}{2}}} = \dfrac{4\sqrt[5]{y^2}y^{\frac{1}{2}}}{x^{-5}x^{\frac{1}{2}}} = \dfrac{4\sqrt[5]{y^2}y^{\frac{1}{2}}}{x^{-\frac{9}{2}}} = 4x^{\frac{9}{2}}\sqrt[5]{y^2}y^{\frac{1}{2}}$

b) $\left(\dfrac{50x^2y^4}{2x^4y^7}\right)^{\frac{1}{2}} = \sqrt{25x^6y^{11}} = 5\sqrt{x^6y^{11}}$

Examples #2

a) A cone with equal height and radius has a volume of 18cm^3. What are the radius and height of the cone to the nearest tenth of a centimetre?

$Vol_{cone} = \frac{4}{3}\pi r^3$

Example #7

b) A hemisphere has a volume of 569m^3. What is the radius of the hemisphere to the nearest tenth of a metre?

$$Vol_{hem} = \frac{2}{3}\pi r^3$$